Bernoulli Distribution

X = obtaining a "success" in experiment with only 2 outcomes – "success", "failure". $P(Success) = p$.

$$X \sim Bern(p)$$

- Probability Mass Function (PMF)

$$p_X(x) = p^x(1-p)^{1-x} \quad \text{for } x = 0, 1$$

- Cumulative Distribution Function (CDF)

$$F_X(t) = P(X \le t) = \begin{cases} 0 & \text{for } t < 0 \\ 1-p & \text{for } 0 \le t < 1 \\ 1 & \text{for } t \ge 1 \end{cases}$$

- Expected Value: $E(X) = p$
- Variance: $Var(X) = p(1-p)$

Binomial Distribution

X = # of "successes" in n trials, where each trial has only 2 outcomes – "success", "failure". $P(Success) = p$.

$$X \sim Bin(n, p)$$

- Probability Mass Function (PMF)

$$p_X(x) = \binom{n}{x} p^x(1-p)^{n-x} \quad \text{for } x = 0, 1, 2, \ldots, n$$

- Cumulative Distribution Function (CDF)

$$F_X(t) = P(X \le t) = \sum_{x=0}^{\lfloor t \rfloor} \binom{n}{x} p^x(1-p)^{n-x}$$

- Expected Value: $E(X) = np$
- Variance: $Var(X) = np(1-p)$

Geometric Distribution

X = # of trials until 1^{st} success where each trial has only 2 outcomes (success, failure). $P(Success) = p$.

$$X \sim Geo(p)$$

- Probability Mass Function (PMF)

$$p_X(x) = P(X = x) = (1-p)^{x-1}p \quad \text{for } x = 1, 2, 3, \ldots$$

- Cumulative Distribution Function (CDF)

$$F_X(t) = P(X \le t) = \begin{cases} 0 & \text{for } t < 1 \\ 1-(1-p)^{\lfloor t \rfloor} & \text{for } t \ge 1 \end{cases}$$

- Expected Value: $E(X) = \frac{1}{p}$
- Variance: $Var(X) = \frac{1-p}{p^2}$

Poisson Distribution

X = # of events occurring during an interval.

$$X \sim Pois(\lambda)$$

- Probability Mass Function (PMF)

$$p_X(x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad \text{for } x = 0, 1, 2, 3, \ldots$$

  where $\lambda > 0$ is the rate parameter.
- Cumulative Distribution Function (CDF)

$$F_X(x) = P(X \le t) = \sum_{x=0}^{\lfloor t \rfloor} p_X(x)$$

- Expected Value: $E(X) = \lambda$
- Variance: $Var(X) = \lambda$