Lecture 12

Uniform Distribution

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Continuous Distributions

Continuous Distributions

Common distributions for continuous random variables

- Uniform distribution

$$X \sim Unif(a, b)$$

- Exponential distribution

$$X \sim Exp(\lambda)$$

- Gamma distribution

$$X \sim Gamma(\alpha, \lambda)$$

- Normal distribution

$$X \sim Normal(\mu, \sigma^2)$$

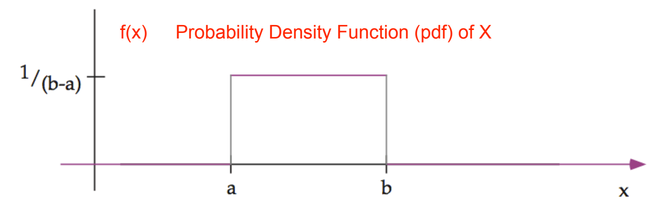
Uniform Distribution

Uniform Distribution

If a random variable follows a *uniform distribution*, then the R.V has constant probability between values a and b.

$$X \sim Unif(a, b)$$

- Probability Density Function (pdf)
  - Im(X) = (a,b)
  - $f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a < x < b \\ 0 & \text{otherwise} \end{cases}$



Uniform Distribution Cont.

- Cumulative Distribution Function (cdf)

$$F_X(t) = \begin{cases} 0 & \text{for } t \leq a \\ \frac{t-a}{b-a} & \text{for } a < t < b \\ 1 & \text{for } t \geq b \end{cases}$$

- Expected Value: $E(X) = \frac{a+b}{2}$

$$E(X) \;=\; \int_a^b \frac{x}{b-a} dx = \frac{1}{b-a}\left(\frac{x^2}{2}\right)\Big|_a^b = \frac{b^2-a^2}{2(b-a)} = \frac{a+b}{2}$$

- Variance: $Var(X) = \frac{(b-a)^2}{12}$

$$Var(X) \;=\; \int_a^b \left(x - \frac{a+b}{2}\right)^2 \frac{1}{b-a} dx = \ldots = \frac{(b-a)^2}{12}$$

Can also get variance by $Var(X) = E(X^2) - [E(X)]^2$

Example

Uniform Distribution Example

Example 1: A basic (pseudo) random number generator creates realizations of $\text{Unif}(0, 1)$ random variables.
X = number obtained from the random number generator.

1. What is $\text{Im}(X)$?
2. Give the pdf and cdf of X

Uniform Distribution Example

3. What is the probability that it generates a number greater than 0.85?

Uniform Distribution Example

3. What is the probability that it generates a number between 0.1 and 0.85?

4. What is the expected value?

5. What is the variance?

Uniform Distribution Example

Example 2: Suppose X has a uniform distribution between 5 and 10. Calculate

1. $P(X < 7) =$
2. $P(6 < X < 7) =$

Uniform Distribution Example