Lecture 19

Introduction to Statistics

Manju M. Johny

STAT 330 - Iowa State University

Terminology

Population and Sample

Population: All individuals (or items) of interest.

- Typically, we want to learn *something* about the population
- Usually impossible to get information from entire population

Sample: A subset of the population

- Since samples are much smaller than population, it possible to actually get information about the sample
- This information is called *"data"* or *"sample data"*.

Statistics: (as a field)

- Use probability to learn about the real world (population) from data (sample).
- Assuming random mechanism generated the data allows us to use probability.

Random Variables and Observations

Population "measurements":

- $X_1, X_2, \ldots, X_n \overset{iid}{\sim} f_X(x)$
- X_i represents (theoretical) measurements from the population.
- $f_X(x)$ represents the population distribution.

Observations:

- $x_1, x_2, \ldots, x_n$ are the observed data (or realization of the random variables).
- x_i are actual measurements from the sample.

Use the observed values $(x_1, \ldots, x_n)$ to learn about the population.

Example

Example 1: A machine fills bottles of water. We are interested in the amount of water filled in the bottles.

- X_i = amount of water filled in bottle i for $i = 1, \ldots, n$
- X_i follows *some* distribution (with *some* parameters).
- But, its impossible to measure *every* bottle that the machine fills

So take a *sample* of n bottles from the machine, and measure the amount of water in them.

- Gives observed values $x_1, \ldots, x_n = (500.01, \ldots, 499.80,)$
- Use this information to understand how much water the machine fills in general (population)

Drawing Samples

Typically, we assume a *simple random sample (SRS)* is drawn from the population to create our sample

- All subsets of same size are equally likely to be chosen
- Guarantees the sample is representative of population
  → leads to good inferences
- If not, we will introduce *bias* in our sample
  → inferences can be way off from the truth
  → leads to untrustworthy results

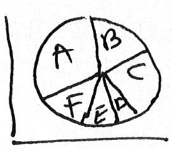
Descriptive Statistics

Descriptive Statistics

Once we have obtained data from the sample, what comes next?

Descriptive Statistics: Describe/summarize key features of the data

- Graphics → visualize the data, describe shape, etc



- Numbers → numerical summaries of quantities of interest

" Typical value" median $\bar{x}$

how spread apart is my data? range = max - min s

No conclusions are made yet. We just want an idea of what the data looks like.

Inference

Estimation

Inferential Statistics: Draw conclusions about the population/distribution that generate the data.

1. Estimation: Estimate the parameters of the probability distribution that generated the data

   - In probability portion of the course, we assumed we knew the parameters of the distribution to answer questions

     - Ex: Get average of 5 hits per hour to a website. $X = \#$ of hits in next hour. $X \sim Pois(5)$. What is $P(X < 3)$?

   - In statistics, parameters are unknown and need to be estimated by the data

     - Confidence intervals
     - Hypothesis testing

Prediction

2. Prediction: Estimate parameters of a data model, then use model to predict values for new observations

Example 2: X = ACT score; Y = Freshman GPA

We model relationship as between X and Y as:

$$Y = f(X) + \epsilon$$

Use data to learn about the form of $f(X)$, "fit" a model, and then we can predict the GPA of a new student based on their ACT score.

$$\hat{Y}_{new} = \hat{f}(x)$$